Characterization of a Radiation-Pressure-Driven Micromechanical Oscillator

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Abstract-**We present results of an experimental study of the oscillation frequency, linewidth, RF-spectrum and the phase noise of a radiation-pressure-driven optomechanical oscillator in a microtoroidal geometry. Through this study we identify the critical parameters that can be used for tailoring the desired characteristics of this device.**

I. INTRODUCTION

We have recently demonstrated a new class of RF oscillator functioning in the optomechanical domain and acquiring gain directly from a continuous wave optical field [1]. This device may be considered as the photonic counterpart of a quartz oscillator, but where the electrical stimulation is replaced by optical stimulation and an inherent feedback mechanism takes over the task of the external feedback circuitry. In this work, we experimentally investigate the oscillation frequency, phase-noise and the RF spectrum of the reported optomechanical oscillations. Fig. 1 shows a device schematic, which is in the form of a silica microtoroid. The toroidal structure supports radio-frequency vibrational modes ($\Omega \sim$10-100 MHz) and provides optical confinement for photons. The toroid optical modes exhibit *Q*-factors in excess of 100 million [2], leading to large circulating powers with only minute coupled optical power. The radiation pressure associated with this large circulating optical power induces the optomechanical coupling responsible for regenerative mechanical oscillation (threshold of a few microwatts). Mechanical oscillation manifests itself as amplitude modulation of the transmitted optical power.

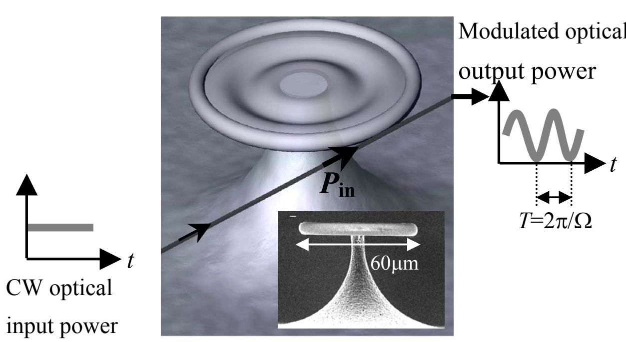


Fig. 1. The schematic diagram illustrating the excitation of the third vibrational mode of the microtoroid through radiation pressure (the mechanical deformation is exaggerated; the actual displacement is on the order of 1A°). The inset is an SEM picture of the actual device.

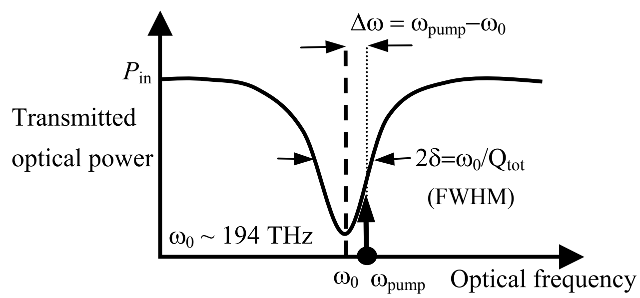


Fig. 2. Schematic diagram of optical transmission spectrum near an optical resonance.

We explore the impact of optical-frequency-detuning, $\Delta\omega$ (Fig. 2), and optical input power on oscillation amplitude, spectral purity, and phase noise performance of the optomechanical oscillator. These measurements are an essential step towards optimization and possible application of the optomechanical oscillator as an all-optical oscillator for optical communication and photonic signal processing applications. Fig. 2 summarizes the definition of the important optical parameters in this system.

II. OPTOMECHANICAL OSCILLATOR

A. *Oscillation amplitude*

When $P_{in}>P_{th}$ (P_{th}: optomechanical oscillation threshold power [1]) the regenerative optomechanical oscillation at the fundamental mechanical frequency (Ω) begins. Since the optical transfer function of an optical resonator has a Lorentzian shape, modulation of the optical resonance results in nonlinear amplitude modulation of the transmitted optical power. This nonlinear transfer function manifests itself through the appearance of harmonics of the fundamental frequency (i.e. 2Ω, 3Ω,…) within the optical power modulation spectrum. Since P_{th} is a function of $\Delta\omega$ the oscillation amplitude (of each harmonic) also depends on $\Delta\omega$. At an optimal $\Delta\omega$ (where oscillation amplitude is maximized) oscillation amplitude is proportional to P_{in}-P_{th} and as P_{in} increases reaches a saturated amplitude due to the limited optical bandwidth [1].

B. *Oscillation frequency*

The passive (intrinsic) mechanical oscillation frequency (Ω_0') of the microtoroidal structure is determined by the geometry, temperature as well as the ambient pressure.

Above threshold when the regenerative optomechanical oscillation starts, the oscillation frequency (Ω) also depends on the circulating optical power (and therefore P_{in}) as well as $\Delta\omega$. The circulating optical power and the frequency detuning ($\Delta\omega$) modify the intrinsic resonant frequency (Ω_0') through optical spring effect [3] and optical absorption. Optical absorption generates a temperature gradient in the structure while the optical spring effect affects the stiffness of the structure by introducing a radiation pressure induced radial force. These effects can be summarized in a single coefficient (η_p). The optical power, optical detuning and temperature dependence of the optomechanical oscillations can be summarized as:

$$\Omega = \Omega_0'(\boldsymbol{T})\left[1+\boldsymbol{\eta_P}(\Delta\boldsymbol{\omega})\boldsymbol{P_{in}}\right] \tag{1}$$

$$\Omega_0'(\boldsymbol{T}) = \Omega_0\left[1+\boldsymbol{\eta_T}(\boldsymbol{T}-\boldsymbol{T}_0)\right] \tag{2}$$

$\eta_p(\Delta\omega)$ and η_T are determined by the optical and mechanical properties of the microtoroid as well as its geometry.

C. Oscillation phase noise and linewidth

The low frequency phase noise of the device is an indicator of the relatively slow noise mechanisms in the system. These mechanisms are generally complicated and hard to identify. Since the optical detuning is not locked in this study and the optical coupling is achieved through a fiber-taper (with low mechanical stability) the separation of the fundamental mechanisms is not possible in this regime. So we only focus on high-frequency regime that carries the signature of the oscillation linewidth.

Above threshold ($P_{in} > P_{th}$) where the mechanical loss is canceled out by the optomechanical gain, the oscillation linewidth follows the general inverse power line narrowing principle in oscillators. In a thermally limited oscillator this theory can be written as [5]:

$$\Delta\Omega = \frac{k_B T}{2P_d}(\Delta\Omega_0)^2 \tag{3}$$

Where P_d is the oscillator output power. In a mechanical oscillator this relation translates to an inverse quadratic relation between the oscillation frequency and oscillation amplitude [6]. For a microtoroidal optomechanical oscillator this relation can be restated as:

$$\Delta\Omega = \left(\frac{4\mu k_B T Q_{tot}^2}{m_{eff}\Omega_0^2 R_0^2}\right)\frac{\Gamma_\Omega^2\Delta\Omega_0}{M^2} \tag{4}$$

Where Q_{tot} is the total optical quality factor, T is the temperature, m_{eff} is the effective mass of the corresponding mechanical mode, k_B is the Boltzman constant, R_0 is the principal radius of the microtoroid, Γ_Ω is the bandwidth coefficient, M is the optical modulation depth and μ is the radial correction factor for the mechanical motion [7]. So if the thermal noise (Brownian noise in the mechanical structure) is the dominant noise mechanism in the oscillator, then the oscillation linewidth should follow equation 4.

III. EXPERIMENTAL RESULTS

Fig. 3 shows the schematic diagram of the experimental arrangement. P_{in} and $\Delta\omega$ are controlled using a tunable laser and the detected optical power is analysed using a RF spectrum analyser and a phase noise analyser.

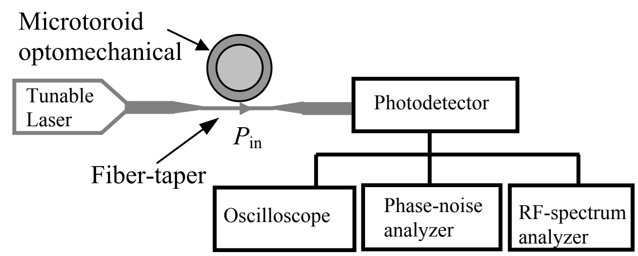


Fig. 3. Schematic diagram of the experimental arrangement

The silica microtoroid studied in this work (inset Fig. 1) has a principal diameter of 60 μm and a pillar diameter of 5μm at the silica-silicon junction. The optical modes of this microtoroid had intrinsic quality factor (Q_0) between 3×10^6 and 7×10^6. The first four mechanical modes of this microtoroid had resonant frequencies of 2.4 MHz, 15.92 MHz, 39.64 MHz, and 54.18 MHz respectively. Due to its low threshold power the 4th flexural mechanical mode with a measured mechanical quality factor (Q_{mech}) of about 2100 has been chosen for this study. For an optical mode with $Q_0 = 5.5\times10^6$ and optimized optical coupling the measured optical threshold power (P_{th}) was about 250 μW.

Fig. 4(a) shows the detected RF power at the fundamental frequency and its harmonics plotted against relative optical frequency detuning from resonance (optical input power is $2\times P_{th}$ = 500 μW). The maximum optical modulation depth (corresponding to the maximum optomechanical oscillation amplitude) occurs when the relative optical frequency detuning ($\Delta\omega/2\delta$) is between 0.5 and 0.6. Fig. 4(b) shows the detected RF power at the fundamental mechanical eigen frequency and its harmonics plotted against relative optical input power to the microtoroid ($\Delta\omega$ is set to its optimal value at each power level). The 2nd and 3rd harmonic suppression ratios are 20dB and 35dB at $1.2\times P_{th}$ and they gradually decrease to 7dB and 17 dB at $2\times P_{th}$.

Fig. 5(a) shows the measured optomechanical oscillation frequency (Ω) plotted against optical frequency detuning, for two different optical quality factors. The value of Q_{tot} is tuned by adjusting the gap between the fiber-taper and the microtoroid (i.e., adjusting the optical coupling coefficient). Note that due to different values of $Q_{tot,}$ the threshold optical power for self-sustained oscillations (P_{th}) is also different in each coupling regime. The solid lines are the calculated one-parameter theoretical fits obtained from equation 1.

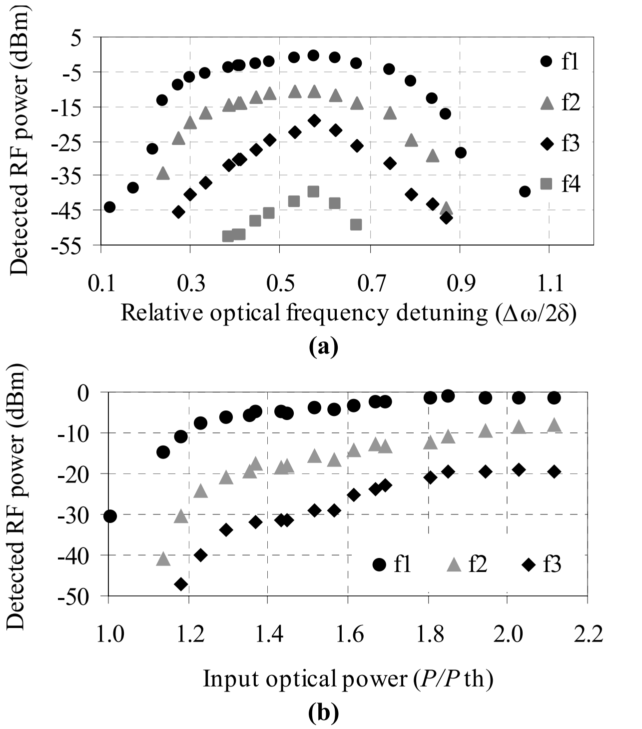


Fig. 4. (a) Measured RF power at the fundamental oscillation frequency (f_1= 54.2 MHz) and its second, third and fourth harmonics plotted against relative optical frequency detuning from resonance (optical input power was $2\times P_{th}$ = 500 μW). (b) Measured RF power at fundamental oscillation frequency (f_1) and its harmonics plotted against relative optical input power. Δω is kept at its optimum value during the measurement.

Fig. 5(b) shows the measured oscillation frequency plotted against the resonator temperature. The estimated value of η_T from the linear fit (the solid line) is about 6.5 KHz/°K (equation 2). Note that unlike the frequency shift generated by optical absorption, this thermal frequency shift is independent of the circulating optical power. For the optomechanical oscillator presented in this work $\eta_T \sim 1.3\times10^{-4}$ K^{-1},$\eta_p(\delta) \sim 1.5\times10^{-6}$ μW^{-1} and $d\eta_p/d(\Delta\omega) \sim 4.8\times10^{-3}$ $\mu W^{-1}pm^{-1}$. These coefficients may be used in equation 1 and 2 to estimate the sensitivity of the oscillation frequency to optical power, ambient temperature and optical frequency detuning.

Fig. 6 shows a typical result of measurement of the phase noise spectrum of the fourth mechanical mode. The inset shows microtoroid deformation at the fourth flexural mechanical mode. We observe a dependence of $1/f^3$ at offset frequencies (Δf) between 100 Hz and 10 KHz and a dependence of $1/f^2$ at offsets larger than 10 KHz. The observation of these two regimes is in good agreement with general phase noise behavior of closed-loop oscillators [4]. The $1/f^3$ regime is a signature of $1/f$ or "flicker noise" in the system while the appearance of $1/f^2$ regime is mainly due to the presence of the white noise. Fig. 7(a) shows the phase noise of the optomechanical oscillator measured at 10^5 Hz offset from the carrier frequency (~54.2 MHz), plotted against relative optical frequency detuning at 500μW ($2\times P_{th}$) optical input power.

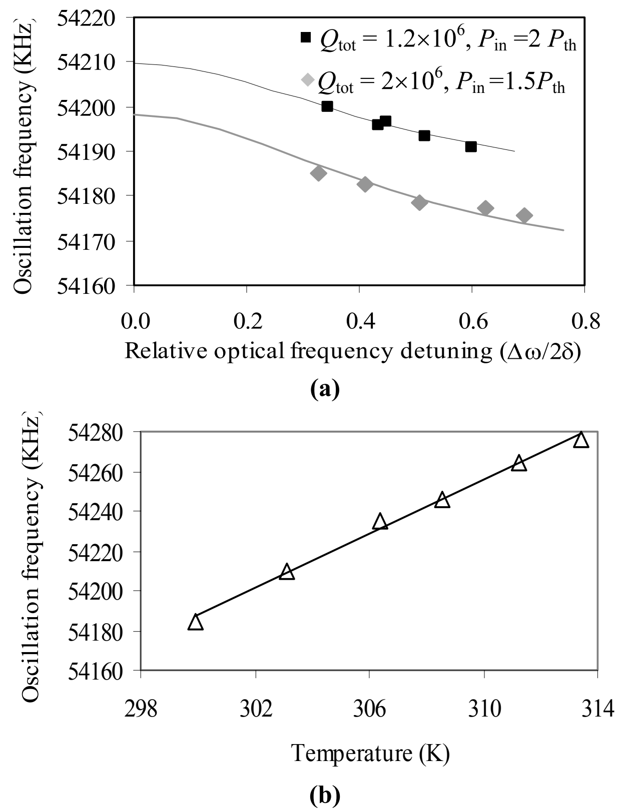


Fig. 5. (a) Mechanical oscillation frequency (Ω) plotted against optical frequency detuning at above threshold regime. The solid lines are theoretical fits obtained from equation 1. (b) Measured oscillation frequency (Ω) plotted against the resonator temperature. The solid line is a linear fit to the experimental data.

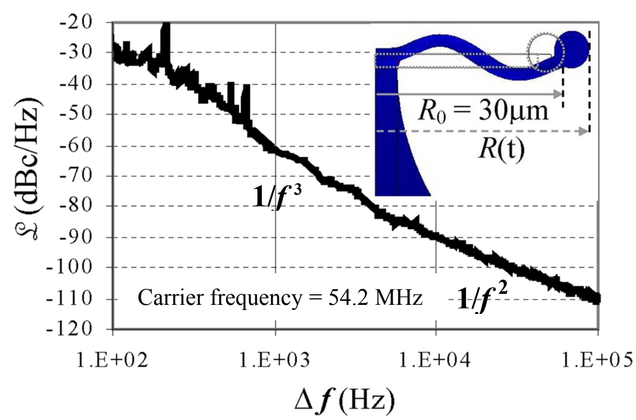
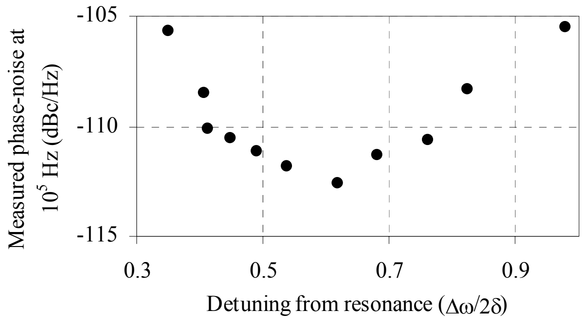
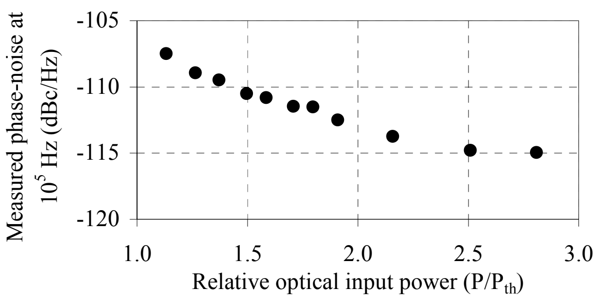


Fig. 6. A typical phase noise spectrum of the optomechanical oscillation. The inset shows the finite element modeling of the 4th flexural mechanical eigen mode. The deformation is exaggerated to show deviation from equilibrium (dotted line)

Fig. 7(b) shows the phase noise of the optomechanical oscillator measured at 10^5 Hz offset from the carrier frequency and for optical detuning $\Delta\omega/2\delta \approx 0.5$, plotted against relative optical input power. As evident from the figure phase noise decreases with the optical power. Since, in the $1/f^2$ regime, the phase noise spectral density is proportional to the short-term linewidth $\Delta\Omega$ [4], this behavior shows that $\Delta\Omega$ varies inversely with the oscillation amplitude (the precise dependence being inverse quadratic). This short-term linewidth amplitude dependence links the source of phase noise to fundamental noise.



(a)



(b)

Fig. 7. (a) Phase noise of the optomechanical oscillator measured at 10^5 Hz offset from the carrier frequency (54.2 MHz) and plotted against relative optical frequency detuning. Optical input power is about $2\times P_{th}$ = 500μW. (b) Phase noise of the optomechanical oscillator measured at 10^5 Hz from the carrier frequency at $\Delta\omega/2\delta \approx 0.5$ and plotted against relative optical input power

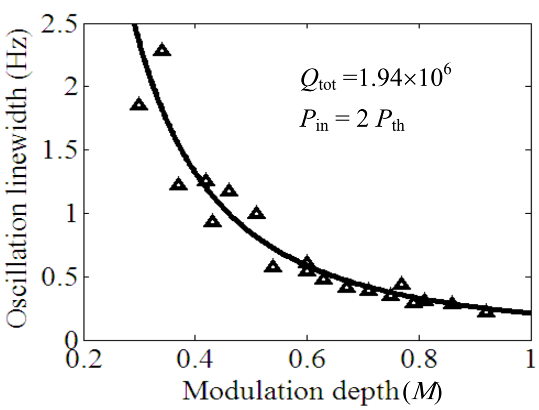


Fig. 8. Measured oscillation linewidth plotted versus modulation depth (M). The solid line is the calculated linewidth using equation 4.

These results clearly show that oscillation linewidth varies inversely with the mechanical energy stored in the oscillator. Although this is in agreement with the general theory of line narrowing in a self-sustained oscillator, it does not yet reveal the fundamental limit for oscillation linewidth. However validation equation 4 is a necessary and sufficient condition for identifying the thermal noise as the fundamental limit for the oscillation linewidth. Fig. 8 shows the measured oscillation linewidth versus optical modulation depth (M). The oscillation linewidth is extracted from the $1/f^2$ regime in the phase noise spectrum. The solid line is the theoretical prediction obtained from equation 4 ($m_{eff} \sim 2.3\times10^{-11}$ Kg, $\Delta\omega \approx \delta$, $\Gamma_\Omega \sim 0.55$, Q_{mech} = 2090 and T = 300 K). The good agreement between theoretical prediction and the experimental result proves that the fundamental limit for the linewidth of a microtoroid optomechanical oscillator is actually the thermal noise.

IV. CONCLUSION

We have studied the effect of the optical frequency detuning optical input power and temperature on the frequency spectrum, phase noise, oscillation linewidth and oscillation frequency of a microtoroidal optomechanical oscillator. The outcome of our measurements shows that for a typical microtoroid resonator, and assuming an optical pump power of about $1.5P_{th}$ and a detuning of about half of the optical linewidth (δ), optomechanical oscillation has a relatively clean spectrum (>10 dB second harmonic suppression ratio) with an oscillation linewidth in sub hertz regime. Our measurements have also confirmed that the fundamental limit of the oscillation linewidth is set by the presence of Brownian noise in the optomechanical oscillator.

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